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Light-front Hamiltonian and path integral formulations of large N scalar QCD_2

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ABSTRACT

Recently Grinstein, Jora and Polosa (2009) [5] have studied a model of large N scalar quantum chromodynamics (QCD) in one-space one-time dimensions (cf. G. 't Hooft (1974) [6]). This theory admits a Bethe–Salpeter equation describing the discrete spectrum of $q\bar{q}$ bound states. They consider the gauge fields in the adjoint representation of $SU(N)$ and the scalar fields in the fundamental representation. The theory is asymptotically free and linearly confining. In this work, we present the light-front quantization of this theory using the Hamiltonian and path integral formulations under appropriate light-cone gauges.

1. Introduction

Very recently 't Hooft et al. [1] and others [2–4], have shown how one could explain the decays of the light scalar mesons by assuming a dominant diquark–antidiquark ($Q\bar{Q}$) structure for the lightest scalar mesons, where the diquark (Q) is being taken to be a spin zero antitriplet color state [1–4]. It is important to emphasize here that in the first approximation, the nonet formed by $f_0(980)$, $a_0(980)$, $\kappa(900)$, $\sigma(500)$ is interpreted as the lowest $Q\bar{Q}$ multiplet [1–4], and the decuplet of scalar mesons with masses above 1 GeV, formed by $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(1450)$, $K_0(1430)$ and possibly containing the lowest glueball, is interpreted as the lowest $q\bar{q}$ scalar multiplet (cf. Refs. [1–4]). The work of Grinstein et al. [5] is seen to further support this hypothesis. Further, Grinstein et al. [5] have studied a model of large N scalar quantum chromodynamics (QCD) [1–7] in one-space one-time dimensions. Their model admits [5] a Bethe–Salpeter equation describing the discrete spectrum of $q\bar{q}$ bound states [1–7].

In the work of Grinstein et al. [5], the gauge fields have been considered [5] in the adjoint representation of $SU(N)$ and the scalar fields in the fundamental representation. The theory is asymptotically free and linearly confining [5]. Different aspects of this theory have been studied by several authors in various contexts [1–10]. The LF theory is seen to be gauge-invariant (GI) possessing a set of first-class constraints.

In the present work, we quantize [11–16] this theory on the light-front (LF) (i.e., on the hyperplanes defined by the equal light-cone (LC) time $\tau = x^+ = (x^0 + x^1)/\sqrt{2} = \text{constant}$ [15,17,18], using the Hamiltonian [11–16] and path integral [12–16] formulations under appropriate LC gauge-fixing conditions (GFC's).

The discretized LC quantization (DLCQ) [18,19] and the basis LFQ (BLFQ) [20] of this theory, where we wish to make contact with the experimentally observational aspects of this theory, are presently under investigation and would be reported later in a separate paper [8]. LF quantization (LFQ) of this theory is presented in the next section using the Hamiltonian and path integral formulations and the summary and discussion is given in Section 3.

2. LF Hamiltonian and path integral formulations

In this section we study the LF Hamiltonian and path integral formulations of a 2D model of large N scalar QCD studied by 't Hooft et al. [1] under appropriate LC gauge-fixing. The bosonized action of the theory that we propose to study is defined (suppressing the color indices) by the action:

$$S = \int \mathcal{L}(\phi, \phi^\dagger, A^\mu)\, d^2x$$

$$\mathcal{L} = \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \partial_\mu \phi^\dagger \partial^\mu \phi - \lambda \left[A_\mu \phi^\dagger \partial^\mu \phi + A^\mu \phi \partial_\mu \phi^\dagger \right] - m^2 \phi^\dagger \phi \right]$$

$$\lambda = \frac{g}{\sqrt{N}}, \qquad F^{\mu\nu} = \left(\partial^\mu A^\nu - \partial^\nu A^\mu \right)$$

$$g^{\mu\nu} = g_{\mu\nu} := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \mu, \nu = +, - \tag{1}$$

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In the Lagrangian density of the theory (cf. above equation), the first term represents the kinetic energy of the gluon field (the color indices have been suppressed), the second term represents the kinetic energy term for the scalar (diquark) field, the third term represents the interaction term for the scalar (diquark) field with the gluon field (the color indices have again been suppressed) and the last term represents the mass term for the scalar (diquark) field. Above action of the theory in LF coordinates $x^{\pm} := (x^0 \pm x^1)/\sqrt{2}$ reads:

$$S = \int \mathcal{L}\, dx^+ dx^-$$

$$\mathcal{L} = \bigg[\frac{1}{2}\left(\partial_+ A^+ - \partial_- A^-\right)^2 + \left(\partial_+\phi^\dagger \partial_-\phi + \partial_-\phi^\dagger \partial_+\phi\right) - \lambda A^-\left(\phi^\dagger\partial_-\phi + \phi\partial_-\phi^\dagger\right) - \lambda A^+\left(\phi^\dagger\partial_+\phi + \phi\partial_+\phi^\dagger\right) - m^2\phi^\dagger\phi\bigg] \tag{2}$$

In the work of Ref. [5], the authors have studied the above action described by Eq. (2), however, with one noticeable difference: namely, that they have implemented the gauge-fixing condition $A^+ \approx 0$ "strongly" in Eq. (2). Also there exists a typo error of a minus sign in their interaction term $[-\lambda A^-(\phi^\dagger\partial_-\phi - \phi\partial_-\phi^\dagger)]$ (as it appears in their Eq. (4), cf. Ref. [5]) which should correctly read as $[-\lambda A^-(\phi^\dagger\partial_-\phi + \phi\partial_-\phi^\dagger)]$ as it appears in our above Eq. (2). In the present work we propose to study the theory defined by the above action (2), following the standard Dirac quantization procedure (DQP) [11] (and we do not fix any gauge at this stage). In our present work we consider this GFC ($A^+ \approx 0$) only as one of the gauge constraints [11] which becomes strongly equal to zero only on the reduced hypersurface of the constraints and remains non-zero in the rest of the phase space of the theory and we do not set it strongly equal to zero in Eq. (2) as is done in the work of Ref. [5].

Also, it may be in order to emphasize here one of the salient features of the DQP [11] with respect to the GFC's in particular. In the DQP, the GFC's are infact, the gauge-constraints which like all other constraints of the theory, are also weakly equal to zero in the sense of Dirac [11], and they become strongly equal to zero only on the reduced hypersurface of the constraints of the theory and not in the rest of the phase space of the classical theory (in the corresponding quantum theory these weak equalities become the weak operator equalities). These considerations hold true also for the GFC's considered by us later (namely, $A^+ \approx 0$ and $A^- \approx 0$) and the calculation presented in the present work is fully consistent with the above DQP [11]. In view of the above, in order to be consistent with the DQP [11], one can not set A^+ to be strongly equal to zero in Eq. (2). This is the only difference in our present work with that of the work of Ref. [5] where the authors have set A^+ to be strongly equal to zero in Eq. (2) right from the beginning.

In the following, we will study the LF Hamiltonian and path integral formulations of the theory defined by the action in Eq. (2). The Euler–Lagrange equations of motion of the theory are obtained as:

$$\begin{aligned}
&-\partial_+\partial_+ A^+ + \partial_+\partial_- A^- - \lambda\left(\phi^\dagger\partial_+\phi + \phi\partial_+\phi^\dagger\right) = 0\\
&-\partial_-\partial_- A^- + \partial_-\partial_+ A^+ - \lambda\left(\phi^\dagger\partial_-\phi + \phi\partial_-\phi^\dagger\right) = 0\\
&-m^2\phi^\dagger - \partial_+\left(\partial_-\phi^\dagger - \lambda A^+\phi^\dagger\right) - \partial_-\left(\partial_+\phi^\dagger - \lambda A^-\phi^\dagger\right) = 0\\
&-m^2\phi - \partial_+\left(\partial_-\phi - \lambda A^+\phi\right) - \partial_-\left(\partial_+\phi - \lambda A^-\phi\right) = 0
\end{aligned} \tag{3}$$

In the following, we would consider the Hamiltonian formulation of the theory described by the above action. The canonical momenta obtained from the above action are:

$$\begin{aligned}
\pi &:= \frac{\partial\mathcal{L}}{\partial(\partial_+\phi)} = \left(\partial_-\phi^\dagger - \lambda A^+\phi^\dagger\right)\\
\pi^\dagger &:= \frac{\partial\mathcal{L}}{\partial(\partial_+\phi^\dagger)} = \left(\partial_-\phi - \lambda A^+\phi\right)\\
\Pi^+ &:= \frac{\partial\mathcal{L}}{\partial(\partial_+ A^-)} = 0\\
\Pi^- &:= \frac{\partial\mathcal{L}}{\partial(\partial_+ A^+)} = \left(\partial_+ A^+ - \partial_- A^-\right)
\end{aligned} \tag{4}$$

Here $\pi, \pi^\dagger, \Pi^+$ and Π^- are the momenta canonically conjugate respectively to $\phi, \phi^\dagger, A^-$ and A^+. The above equations however, imply that the theory possesses three primary constraints:

$$\chi_1 = \Pi^+ \approx 0, \qquad \chi_2 = \left[\pi - \partial_-\phi^\dagger + \lambda A^+\phi^\dagger\right] \approx 0$$
$$\chi_3 = \left[\pi^\dagger - \partial_-\phi + \lambda A^+\phi\right] \approx 0 \tag{5}$$

The symbol $\approx$ here denotes a weak equality in the sense of Dirac [11], and it implies that these above constraints hold as a strong equality only on the reduced hypersurface of the constraints and not in the rest of the phase space of the classical theory (and similarly one can consider it as a weak operator equality for the corresponding quantum theory).

The canonical Hamiltonian density corresponding to $\mathcal{L}$ is:

$$\begin{aligned}
\mathcal{H}_c &:= \left[\pi\partial_+\phi + \pi^\dagger\partial_+\phi^\dagger + \Pi^+\partial_+ A^- + \Pi^-\partial_+ A^+ - \mathcal{L}\right]\\
&= \bigg[\frac{1}{2}\left(\Pi^-\right)^2 + \Pi^-\left(\partial_- A^-\right) + m^2\phi^\dagger\phi + \lambda A^-\left(\phi^\dagger\partial_-\phi + \phi\partial_-\phi^\dagger\right)\bigg]
\end{aligned} \tag{6}$$

After including the primary constraints χ_1, χ_2 and χ_3 in the canonical Hamiltonian density $\mathcal{H}_c$ with the help of the Lagrange multiplier fields u, v and w, the total Hamiltonian density $\mathcal{H}_T$ could be written as:

$$\begin{aligned}
\mathcal{H}_T = \bigg[&\left(\Pi^+\right)u + \left(\pi - \partial_-\phi^\dagger + \lambda A^+\phi^\dagger\right)v\\
&+ \left(\pi^\dagger - \partial_-\phi + \lambda A^+\phi\right)w + \frac{1}{2}\left(\Pi^-\right)^2 + \Pi^-\left(\partial_- A^-\right)\\
&+ m^2\phi^\dagger\phi + \lambda A^-\left(\phi^\dagger\partial_-\phi + \phi\partial_-\phi^\dagger\right)\bigg]
\end{aligned} \tag{7}$$

The Hamilton's equations of motion of the theory that preserve the constraints of the theory in the course of time could be obtained from the total Hamiltonian (and are omitted here for the sake of brevity): $H_T = \int \mathcal{H}_T\, dx^-$. Demanding that the primary constraint χ_1 be preserved in the course of time, one obtains the secondary Gauss-law constraint of the theory as:

$$\chi_4 = \left[\partial_-\Pi^- - \lambda\left(\phi^\dagger\partial_-\phi + \phi\partial_-\phi^\dagger\right)\right] \approx 0 \tag{8}$$

The preservation of χ_2, χ_3 and χ_4, for all times does not give rise to any further constraints. The theory is thus seen to possess only four constraints χ_i (with $i = 1, 2, 3, 4$). The constraints χ_2, χ_3 and χ_4 could however, be combined in to a single constraint:

$$\psi = \left[\partial_-\Pi^- - \lambda\left(\phi^\dagger\pi^\dagger + \phi\pi\right) - 2(\lambda)^2 A^+\phi^\dagger\phi\right] \approx 0 \tag{9}$$

and with this modification, the new *set* of constraints of the theory could be written as:

$$\Omega_1 = \chi_1 = \Pi^+ \approx 0$$
$$\Omega_2 = \psi = \left[\partial_-\Pi^- - \lambda\left(\phi^\dagger\pi^\dagger + \phi\pi\right) - 2(\lambda)^2 A^+\phi^\dagger\phi\right] \approx 0 \tag{10}$$

Further, the matrix of the Poisson brackets among the constraints Ω_i, with $(i=1,2)$ is seen to be a singular matrix implying that the set of constraints Ω_i is first-class and that the theory under consideration is gauge-invariant.

Vector gauge current density of the theory $j^\mu \equiv (j^+, j^-)$ is:

$$j^+ = \left[i\lambda\beta\phi\partial_-\phi^\dagger - i\lambda\beta\phi^\dagger\partial_-\phi + \lambda(\partial_-\beta)\left(\partial_+A^+ - \partial_-A^-\right)\right]$$
$$j^- = \left[i\lambda\beta\phi\left(\partial_+\phi^\dagger - \lambda A^-\phi^\dagger\right) - i\lambda\beta\phi^\dagger\left(\partial_+\phi + \lambda A^-\phi\right) - \lambda(\partial_+\beta)\left(\partial_+A^+ - \partial_-A^-\right)\right] \tag{11}$$

The divergence of the vector gauge current density of the theory could now be easily seen to vanish satisfying the continuity equation: $\partial_\mu j^\mu = 0$, implying that the theory possesses at the classical level, a local vector-gauge symmetry. The action of the theory is indeed seen to be invariant under the local vector gauge transformations:

$$\delta\phi = i\lambda\beta\phi, \qquad \delta\phi^\dagger = -i\lambda\beta\phi^\dagger$$
$$\delta A^- = \lambda\partial_+\beta, \qquad \delta A^+ = \lambda\partial_-\beta \tag{12a}$$
$$\delta\pi = \left[-i\lambda\beta\partial_-\phi^\dagger - i\lambda\phi^\dagger\partial_-\beta + i(\lambda)^2\beta\phi^\dagger A^+ - (\lambda)^2\phi^\dagger\partial_-\beta\right] \tag{12b}$$
$$\delta\pi^\dagger = \left[i\lambda\beta\partial_-\phi + i\lambda\phi\partial_-\beta - i(\lambda)^2\beta\phi A^+ - (\lambda)^2\phi\partial_-\beta\right] \tag{12c}$$
$$\delta u = \lambda\partial_+\partial_+\beta, \qquad \delta v = i\lambda\partial_+(\beta\phi)$$
$$\delta w = -i\lambda\partial_+\left(\beta\phi^\dagger\right) \tag{12d}$$
$$\delta\Pi^+ = \delta\Pi^- = \delta\Pi_u = \delta\Pi_v = \delta\Pi_w = 0 \tag{12e}$$

where $\beta \equiv \beta(x^+, x^-)$ is an arbitrary function of its arguments. In order to quantize the theory using Dirac's procedure we now convert the set of first-class constraints of the theory η_i into a set of second-class constraints, by imposing, arbitrarily, some additional constraints on the system called gauge-fixing conditions (GFC's) or the gauge-constraints. For this purpose, for the present theory, we could choose, for example, the following set of GFC's:

$$\zeta_1 = A^+ \approx 0, \qquad \zeta_2 = A^- \approx 0 \tag{13}$$

Here the gauge $A^+ \approx 0$ represents the LC time-axial or temporal gauge and the gauge $A^- \approx 0$ represents the LC coulomb gauge and both of these gauges are physically important gauges. Corresponding to this gauge choice, the theory has the following set of constraints under which the quantization of the theory could e.g. be studied:

$$\xi_1 = \Omega_1 = \chi_1 = \Pi^+ \approx 0 \tag{14a}$$
$$\xi_2 = \Omega_2 = \psi = \left[\partial_-\Pi^- - \lambda\left(\phi^\dagger\pi^\dagger + \phi\pi\right) - 2(\lambda)^2A^+\phi^\dagger\phi\right] \approx 0 \tag{14b}$$
$$\xi_3 = \zeta_1 = A^+ \approx 0 \tag{14c}$$
$$\xi_4 = \zeta_2 = A^- \approx 0 \tag{14d}$$

The matrix $R_{\alpha\beta}$ of the Poisson brackets among the set of constraints ξ_i with $(i=1,2,3,4)$ is seen to be nonsingular with the determinant given by

$$\left[\left\| det(R_{\alpha\beta})\right\|\right]^{\frac{1}{2}} = \left[\left[\delta'\left(x^- - y^-\right)\right]\left[\delta\left(x^- - y^-\right)\right]\right] \tag{15}$$

The other details of the matrix $R_{\alpha\beta}$ are omitted here for the sake of brevity. Finally, following the DQP, the nonvanishing equal light-cone-time commutators of the theory, under the GFC's: $A^+ \approx 0$ and $A^- \approx 0$ are obtained as:

$$\left[\phi\left(x^+, x^-\right), \pi\left(x^+, y^-\right)\right] = i\delta\left(x^- - y^-\right) \tag{16a}$$
$$\left[\phi^\dagger\left(x^+, x^-\right), \pi^\dagger\left(x^+, y^-\right)\right] = i\delta\left(x^- - y^-\right) \tag{16b}$$
$$\left[\phi\left(x^+, x^-\right), \Pi^-\left(x^+, y^-\right)\right] = -\frac{i}{2}\lambda\phi\epsilon\left(x^- - y^-\right) \tag{16c}$$
$$\left[\phi^\dagger\left(x^+, x^-\right), \Pi^-\left(x^+, y^-\right)\right] = -\frac{i}{2}\lambda\phi^\dagger\epsilon\left(x^- - y^-\right) \tag{16d}$$
$$\left[\pi\left(x^+, x^-\right), \Pi^-\left(x^+, y^-\right)\right] = \frac{i}{2}\lambda\left(\pi + 2\lambda A^+\phi^\dagger\right)\epsilon\left(x^- - y^-\right) \tag{16e}$$
$$\left[\pi^\dagger\left(x^+, x^-\right), \Pi^-\left(x^+, y^-\right)\right] = \frac{i}{2}\lambda\left(\pi^\dagger + 2\lambda A^+\phi^\dagger\right)\epsilon\left(x^- - y^-\right) \tag{16f}$$
$$\left[\Pi^-\left(x^+, x^-\right), \Pi^-\left(x^+, y^-\right)\right] = -2i(\lambda)^2\phi^\dagger\phi\epsilon\left(x^- - y^-\right) \tag{16g}$$
$$\left[\Pi^-\left(x^+, x^-\right), A^+\left(x^+, y^-\right)\right] = 2i\delta\left(x^- - y^-\right) \tag{16h}$$
$$\left[\Pi^-\left(x^+, x^-\right), \phi\left(x^+, y^-\right)\right] = \frac{i}{2}\lambda\phi\epsilon\left(x^- - y^-\right) \tag{16i}$$
$$\left[\Pi^-\left(x^+, x^-\right), \phi^\dagger\left(x^+, y^-\right)\right] = \frac{i}{2}\lambda\phi^\dagger\epsilon\left(x^- - y^-\right) \tag{16j}$$

The first-order Lagrangian density $\mathcal{L}_{I0}$ of the theory is:

$$\begin{aligned}\mathcal{L}_{I0} :&= \left[\pi(\partial_+\phi) + \pi^\dagger\left(\partial_+\phi^\dagger\right) + \Pi^+\left(\partial_+A^-\right) + \Pi^-\left(\partial_+A^+\right) + \Pi_u(\partial_+u) + \Pi_v(\partial_+v) + \Pi_w(\partial_+w) - \mathcal{H}_T\right] \\ &= \left[\frac{1}{2}\left(\Pi^-\right)^2 - v\left(\lambda A^+\phi^\dagger - \partial_-\phi^\dagger\right) - w\left(\lambda A^+\phi - \partial_-\phi\right) - m^2\phi^\dagger\phi - \lambda A^-\left(\phi^\dagger\partial_-\phi + \phi\partial_-\phi^\dagger\right)\right]\end{aligned} \tag{17}$$

In the path integral formulation, the transition to quantum theory is made by writing the vacuum to vacuum transition amplitude for the theory called the generating functional $Z[J_k]$ of the present theory [12–16] under the GFC's: $\zeta_1 = A^+ \approx 0$ and $\zeta_2 = A^- \approx 0$, in the presence of the external sources J_k as:

$$Z[J_k] = \int [d\mu]\exp\left[i\int d^2x\left[J_k\Phi^k + \pi\partial_+\phi + \pi^\dagger\partial_+\Phi^\dagger + \Pi^+\partial_+A^- + \Pi^-\partial_+A^+ + \Pi_u\partial_+u + \Pi_v\partial_+v + \Pi_w\partial_+w - \mathcal{H}_T\right]\right] \tag{18}$$

Here, the phase space variables of the theory are: $\Phi^k \equiv (\phi, \phi^\dagger, A^-, A^+, u, v, w)$ with the corresponding respective canonical conjugate momenta: $\Pi_k \equiv (\pi, \pi^\dagger, \Pi^+, \Pi^-, \Pi_u, \Pi_v, \Pi_w)$. The functional measure $[d\mu]$ of the generating functional $Z[J_k]$ under the above gauge-fixing is obtained as:

$$\begin{aligned}[d\mu] = &\left[\delta'\left(x^- - y^-\right)\delta\left(x^- - y^-\right)\right][d\phi]\left[d\phi^\dagger\right]\left[dA^+\right]\left[dA^-\right][du][dv] \\ &\times [dw][d\pi]\left[d\pi^\dagger\right]\left[d\Pi^-\right]\left[d\Pi^+\right][d\Pi_u][d\Pi_v][d\Pi_w] \\ &\times \delta\left[\Pi^+ \approx 0\right]\delta\left[A^- \approx 0\right]\delta\left[\left(\partial_-\Pi^- - \lambda\left(\phi^\dagger\pi^\dagger + \phi\pi\right) - 2(\lambda)^2A^+\phi^\dagger\phi\right) \approx 0\right]\delta\left[A^+ \approx 0\right]\end{aligned} \tag{19}$$

The LF Hamiltonian and path integral quantization of the theory under the set of GFC's: $A^+ \approx 0$ and $A^- \approx 0$ is now complete.

3. Summary and discussion

't Hooft et al. [1] and others [2–7], have shown how one could explain the decays of the light scalar mesons by assuming a dominant diquark–antidiquark $(Q\bar{Q})$ structure for the lightest scalar mesons, where the diquark (Q) is being taken to be a spin zero antitriplet color state [1–7]. Grinstein et al. [5] have studied a model of large N scalar quantum chromodynamics (QCD) [5] in one-space one-time dimensions. The model of Grinstein et al. [5] admits a Bethe–Salpeter equation describing the discrete spectrum of $q\bar{q}$ bound states [1–7]. In the their work, the gauge fields have been considered [5] in the adjoint representation of $SU(N)$ and the scalar fields in the fundamental representation. The theory is asymptotically free and linearly confining [5].

In the present work, we have studied the LFQ of the large N scalar QCD model in one-space one-time dimensions [1–7] studied recently by Grinstein et al. [5]. They have considered the gauge fields in the adjoint representation of $SU(N)$ and the scalar fields in the fundamental representation. Different aspects of this theory have been studied by several authors in various contexts [1–7]. In the present work, we have quantized this theory on the LF (i.e., on the hyperplanes defined by the equal light-cone (LC) time $\tau = x^+ = (x^0 + x^1)/\sqrt{2} = \text{constant}$ [15,17,18], using the Hamiltonian [11–16] and path integral [12–16] formulations under appropriate LC gauges. The theory is seen to possess a set of first-class constraints, and consequently it is gauge-invariant (GI) possessing a local vector gauge symmetry. The local vector gauge current is seen to be divergenceless. As mentioned in the introduction, the DLCQ [18,19] and the BLFQ [20] of this theory, where we hope to make contact with the experimentally observational aspects of this theory, are presently under investigation and would be reported later in a separate paper [8].

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